TIME VALUE OF MONEY

- The evaluation of capital expenditure/ budgeting proposals involves the comparison between cash outflow and cash inflow .The evaluation of capital expenditure proposals involves the decision to be taken today whereas the flow of funds, either outflow or inflow, may be spread over a number of years.
- As me know that money has time value, therefore both cash outflow and cash inflow should be compared considering time value. This whole concept is known as **‘Time value of money’**.

According to the time value of money concept a rupee today is more valuable than a rupee a year hence. This is because of several reasons which are as follows:-

i. Individuals prefer current consumption to future consumption.

ii. There is always an element of uncertainty attached with future cash flows.

iii. Capital can be employed productively to generate positive return. An investment of one rupee today would grow to (I+ r) times a year hence where r is rate of return on investment.

iv. In an inflationary period, the purchasing power of cash inflows today is greater than that of equivalent sum received a year hence.

v. There may be investment opportunities available if the amount is received today which cannot be exploited if the equipment sum is received after one year.

- For example if Mr. X is given the option that he can receive an amount of Rs 10000 either today or after one year, he will most definitely select the first option. This is because if he receives Rs 10000 today, he can always invest the same in a bank FD @10%p.a. So if the choice is given to him he will like to receive Rs 10000 today or Rs 11000(i.e. Rs 10000 + 1000 as an interest) after one year. If he has to receive Rs 10000 only after one year the real value of the same is something less than Rs 10000 in term of today. This concept is called ‘Times value of money’.

- Many financial problem involves cash flows occurring at different points of time whereas the evaluation is required to be done as on today. Therefore an explicit consideration of time value of money is required. There are two techniques for this purpose:-

1. Compounding

In this the interest is compounded and becomes a part of intial principal at the end of compounding period. Following formula can be used:-

$$A = P\ (I+K)^n$$

2. Discounting

This technique tries to find out the present value of re 1 if received or spent after n years provided that the interest of k can be earned on investment.

$$P = \frac{A}{(I+K)^n}$$

P = present value of sum received or spent.

A = sum received or spent in future.

K = rate of interest.

n = number of years.

Future Value Of Single Amount

Future value of a single amount in computed with the help of following formula:-

$$FV_n = PV\ (1+k)^n$$

Where,

FV_n = Future Value n year hence.

PV = Present Value.

K = Interest rate p.a.

n = no. of years for which compounding in done.

This equation is a basic equation in compounding analysis.

The **factor $(1+K)^n$** is referred to as the **Compounding Value Factor (CVF) or Future Value Interest Factor (CVIF)**.

In case, where the compounding is done more frequently,the future value of cash single amount is compound by following formula:-

$$FV_n = PV\,(1+k/m)^{m*n}$$

Where, m = no. of times compounding is done during a year.

Future Value of Annuity

- An annuity is a series of periodic cash flows of equal amounts .
- When the cash flow occurs at the end of each period, the annuity is called a **regular annuity or a deferred annuity**.
- When the case flows occurs at the beginning of each period the annuity is called **annuity due**.

In general term, the future value of an Annuity is given by the following formula:-

$$FVA_n = A(1+K)^{n-1} + A(1+K)^{n-2} + -------- + A$$

Or

$$A\,(CVAF_n\,,x) = A\,(FVIFn)$$

$$A\left[\frac{(1+K)^{n}-1}{K}\right]$$

Where,

FVA_n = future value of annuity for n periods

K = interest rate per period

n = duration of the annuity

- The term $\left[\frac{(1+K)^{n}\ -\ 1}{K}\right]$ is referred to the future value interest factor for an annuity$(FVIFA)_{k,n}$
- The above equation shows the relationship between FVA_n , A , R and n. From this equation we get,

$$A = FVA_n\left[\frac{k}{(1+K)^{n}-1}\right]$$

The term $\left[\frac{k}{(1+K)^{n}-1}\right]$, inverse of $FVIFA_{r,n}$ is called 'Sinking Fund Factor'.

Present Value of a Single Amount

- The process of discounting used for calculating the present value is simply the inverse of compounding. The present value of a single amount can be readily obtained by manipulating the compounding formula.

$$FV_n = PV(1+K)^{n}$$

$$PV = FV_n\left[\frac{1}{(1+K)^{n}}\right]$$

$$PV_n = FV_n\ (PVIF_{n,k})$$

$\left[\frac{1}{(1+K)^{n}}\right]$ = **Present Value Interest Factor**

Present Value of an Annuity

- The present value of an annuity is simply the sum of the present values of all the inflows of that particular annuity. In general terms, the present value of an annuity can bae expressed as follows:-

$$PVA_n = \frac{A}{(1+K)^1} + \frac{A}{(1+K)^2} + -------- \frac{A}{(1+K)^{n-1}} + \frac{A}{(1+K)^n}$$

$$= A\left[\frac{1}{(1+K)^1} + \frac{1}{(1+K)^2} + -------- \frac{1}{(1+K)^{n-1}} + \frac{1}{(1+K)^n}\right]$$

$$= \boxed{\mathbf{A}\left[\frac{(1+K)^n-1}{k(1+k)^n}\right]}$$

$\left[\frac{(1+K)^n-1}{k(1+k)^n}\right]$ is referred to the **present value Interest Factor of an Annuity** ($PVIFA_{k,n}$) .

- It is simply equal to the product of the future value interest factor for an annuity ($FVIFA_{k,n}$) and the present value interest factor ($PVIFA_{k,n}$).
- Above equation shows the relationship between PVA_n , k, n and A. We can also get:-

$$\mathbf{A = PVA_n\left[\frac{K(1+K)^n}{(1+K)^n - 1}\right]}$$

$\left[\frac{K(1+K)^n}{(1+K)^n-1}\right]$ = inverse of $PVIFA_{k,n}$ is called the **Capital Recovery Factor (CRF).**

Present Value of a Perpetuity

- A perpetuity is an annuity of infinity duration.

$$\boxed{\mathbf{P_\infty = A \times PVIFA_{k,\infty}}}$$

A = Constant annual payment

$$\mathbf{PVIFA_{k,\infty} = \sum_{t=1}^{\infty} \frac{1}{(1+K)^t} = \frac{1}{k}}$$

- So present value of interest factor of perpetuity is simply 1 divided by interest rate.
- The present value interest factor declines as the interest rate rises and as the length of time increases.

<u>Present Value of an Unseen series</u>

$$\mathbf{PV_n = \frac{A_1}{(1+K)} + \frac{A_2}{(1+K)^2} + ----+ \frac{A_n}{(1+K)^n} = \sum_{t=1}^{n} \frac{A_t}{(1+k)^t}}$$

Where,

PV_n = present value of a cash flow stream.

A_t =cash flow occurring at the end of year t.

K = discount rate. n = duration of cash flow stream.

- Sometimes cash flows may have to be discounted more frequently than once in a year-semi-annually, quarterly or monthly. The shorter discounting period implies that,

(i) No. of periods in the analysis increases

(ii) The discount rate applicable per period decreases.

- The general formula for calculating the present value in the case of shorter discounting period is :-

$$\mathbf{PV = FV_n \left[\frac{1}{(1+\frac{k}{m})}\right]^{m\times n}}$$

m = no. of times discounting is done in a year.